Information Aggregation and Social Networks: Responsiveness and Overturning

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Introduction

Motivation

- When making decisions under uncertainty, individuals often **learn from others**.
  - For example, people rely on others' choices when adopting a new technology.
- What individuals learn from others depends on **whom they can observe**.
- Digitization expands access to others' behavior and reshapes **observation networks**.
- Experimental evidence further illustrates the practical relevance of social observation.
  - Social influence can weaken the wisdom of crowds (Lorenz *et al.*, 2011; Muchnik *et al.*, 2013).
  - Yet, social observation can improve collective accuracy (Becker *et al.*, 2017; Cai *et al.*, 2009).
- **How do network structures affect information aggregation?**

Model

- There are $N \geq 3$ **agents**, where agent i chooses $a_i \in A = \{0, 1\}$ in period i.
- There are binary **states** $\theta \in \Theta = \{L, H\}$ with **prior** $\mu_i \in (0, 1)$ for each i.
- Utility: $u(0, L) = u(1, H) = 1$, and the payoff is zero otherwise.
- In each period i, agent i observes the **history** $(a_j)_{j \in \mathcal{N}_i}$ and her **private signal** s_i.
  - **Network** $\mathcal{N} = (\mathcal{N}_i)_{i=1}^N$: $\mathcal{N}_i \subseteq \{1, \ldots, i-1\}$ specifies agent i's neighborhood.
  - Private signal s_i is (independently) drawn from the **information structure** $\Pi_i = (S_i, \pi_i)$.
  - (μ, Π) denotes the **informational environment**.

LITERATURE

- Social learning/observational learning: (see Bikhchandani *et al.* (2024) for a survey)
  - Pioneering work: Banerjee (1992), Bikhchandani *et al.* (1992), and Smith and Sørensen (2000).
- Social learning in social networks: (see Golub and Sadler (2016) for a survey)
  - Gale and Kariv (2003), Çelen and Kariv (2004), Acemoglu *et al.* (2011), Lobel and Sadler (2015), Mossel *et al.* (2015), Arieli and Mueller-Frank (2019, 2021), Dasaratha *et al.* (2023), Kartik *et al.* (2024).
  - Sgroi (2002): "Guinea pigs" can increase social welfare.
  - Dasaratha and He (2019): Comparisons of information loss due to confounding.
- Non-Bayesian frameworks:
  - Ellison and Fudenberg (1993, 1995), Bala and Goyal (1998, 2001), DeMarzo *et al.* (2003), Golub and Jackson (2010, 2012), Jadbabaie *et al.* (2012), and Molavi *et al.* (2018)

Main Analysis

Evaluation Criterion

- We evaluate network $\mathcal{N}$ by the **focal agent's payoff** $V_N(\mathcal{N}, \mu, \Pi)$ (or social welfare).
  - In a finite society, agent N may correspond to the ultimate decision maker.
  - In an infinite society, N represents a finite learning horizon.
- When signals (not actions) are observed, the predecessor network does not matter.
  - When signals are observable, $\mathcal{N}_N = \mathcal{N}'_N$ implies the same payoff.
- When actions (not signals) are observed, the predecessor network affects inference.

Rationalizable Networks

- $\widehat{\mathcal{N}}$ is **rationalizable** if, for some (μ, Π), $V_N(\widehat{\mathcal{N}}, \mu, \Pi) > V_N(\mathcal{N}, \mu, \Pi)$ for every $\mathcal{N} \neq \widehat{\mathcal{N}}$.

Theorem 1

$\widehat{\mathcal{N}}$ is rationalizable if and only if $\widehat{\mathcal{N}}_N = \{1, \ldots, N-1\}$.

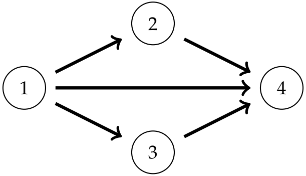
- Necessity: an additional observation cannot hurt agent N.
- Sufficiency: **any** predecessor network **can be** uniquely optimal.
- **Thus, network efficiency depends critically on the informational environment.**

Corollary 1: Social Value of Connectivity

- Fix a link $j \to i$ among predecessors, where $j < i < N$.
- There is an informational environment in which **adding this link increases** V_N.
- There is an informational environment in which **removing this link increases** V_N.
    - The link is weakly valuable to agent i, who gains an observation.
    - But it changes how agent i combines private information and social observation.
    - Her action may therefore become more or less informative to later agents.
- Thus, **connectivity has an ambiguous social value** despite its positive private value.

Proof Sketch of Theorem 1

- Necessity is immediate, and the substantive challenge is the sufficiency direction.
  - One environment must **uniformly** favor the target network over **all** alternatives.



Target shield network

1. Choose **priors** for indifference at the target observation.
2. Give agent 2 and 3 two weak **probe signals**.
   - Only the target neighborhood & history separate them.
3. Give agent 4 **test signals** that make separation valuable.
4. Use further smaller perturbations at each step.

SOCIALLY RATIONALIZABLE NETWORKS

- Fix weights $\lambda = (\lambda_1, \ldots, \lambda_N) \in \mathbb{R}^N_{++}$: define $W_\lambda(\mathcal{N}, \mu, \Pi) = \sum_{i=1}^N \lambda_i V_i(\mathcal{N}, \mu, \Pi)$.
  - For example, $\lambda_i = \delta^{i-1}$ for a discounted sum.
- $\widehat{\mathcal{N}}$ is **socially rationalizable** if $\exists(\mu, \Pi)$, $W_\lambda(\widehat{\mathcal{N}}, \mu, \Pi) > W_\lambda(\mathcal{N}, \mu, \Pi)$, $\forall \mathcal{N} \neq \widehat{\mathcal{N}}$.

Proposition 1

$\widehat{\mathcal{N}}$ is socially rationalizable if and only if $\widehat{\mathcal{N}}_N = \{1, \ldots, N-1\}$.

- The characterization is unchanged under positive welfare weights.

Two Benchmark Networks

- A general construction behind Theorem 1 cannot isolate underlying mechanisms.
  - We want to identify the forces behind the ambiguous social value of connectivity.
- We revisit two networks at opposite ends of connectivity.
  - **Star network**: $\mathcal{N}_i^S = \emptyset$ for $i < N$ and $\mathcal{N}_N^S = \{1, \ldots, N-1\}$.
  - **Complete network**: $\mathcal{N}_i^C = \{1, \ldots, i-1\}$ for every i.
- From now on, agents share a prior of $1/2$ and a common information structure Π.

Star Network

Theorem 2

There exists Π^B such that $V_N(\mathcal{N}^S, \Pi^B) > V_N(\mathcal{N}, \Pi^B)$ for every $\mathcal{N} \neq \mathcal{N}^S$.

- Under binary signals, an isolated agent's action perfectly reveals her signal.
  - Agent N infers every predecessor's private signal under $\mathcal{N}^S$.
- Links among predecessors can immediately generate information cascades.
  - A connected agent may follow history and ignore her private signal.
  - Her action then becomes less responsive and less informative.
- The star network preserves **responsiveness** of predecessors' actions.

Complete Network

Theorem 3

There exists Π^* such that $V_N(\mathcal{N}^C, \Pi^*) > V_N(\mathcal{N}, \Pi^*)$ for every $\mathcal{N} \neq \mathcal{N}^C$.

- Four signals: l, h (ordinary) & l^*, h^* (strong), where l, l^* favor L and h, h^* favor H.
- Under $\mathcal{N}^C$, successive action reversals reveal stronger signals.
  - A history of action 1 is likely generated by the ordinary signal h.
  - A subsequent action 0 indicates the more informative negative signal l.
  - A further action 1 indicates still stronger good news, namely h^*.
  - A final action 0 similarly indicates the strongest bad news l^*.
- Complete observation makes each **overturning** publicly interpretable.

IMPLICATIONS

- The **responsiveness effect** favors sparse networks.
  - Fewer observations weaken informational interdependence among actions.
  - Keeping actions responsive to private signals mitigates cascades.
- The **overturning effect** favors dense networks.
  - Rich public histories identify signals that overturn previous actions.
  - Overturning corrects histories that otherwise induce incorrect inference.
- This trade-off determines whether sparse or dense networks perform better.

Responsiveness and Overturning

Framework

- **Which features of information structures determine which effect dominates?**
- M agents (**guinea pigs**) act independently; later agents observe all preceding actions.
  - For $M \in \{1, \ldots, N-1\}$, $\mathcal{N}_i^M = \emptyset$ for $i \leq M$ and $\mathcal{N}_i^M = \{1, \ldots, i-1\}$ for $i > M$.
  - As M increases, $\mathcal{N}^M$ becomes sparser, from complete ($M = 1$) to star ($M = N - 1$).
- Let $\Pi^{\varepsilon,p} = (\{l^*, l, h, h^*\}, \pi)$ for $p \in (1/2, 1)$ and $\varepsilon \in (0, 1)$:

$$\begin{aligned} \pi(l \mid L) &= \pi(h \mid H) = (1-\varepsilon)p, & \pi(h \mid L) &= \pi(l \mid H) = (1-\varepsilon)(1-p), \\ \pi(l^* \mid L) &= \pi(h^* \mid H) = \varepsilon, & \pi(h^* \mid L) &= \pi(l^* \mid H) = 0, \end{aligned}$$

  - Ordinary signals l, h have accuracy p; conclusive signals l^*, h^* occur with probability ε.
- We derive the number of guinea pigs M^* maximizing agent N's payoff $V_N(\mathcal{N}^M, \Pi^{\varepsilon,p})$.

The Optimal Number of Guinea Pigs

Proposition 3

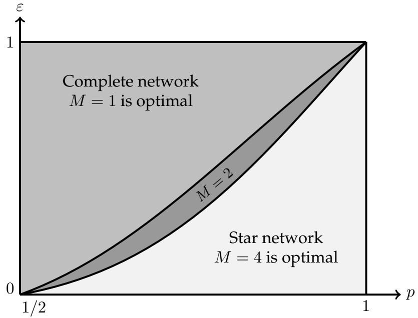
Suppose that $N \geq 3$ is odd. Then, there are unique cutoffs $1 > \varepsilon_0(p) > \varepsilon_1(p) > \cdots > 0$. For $1 \leq k < (N-1)/2$, the optimum is $M^* = 2k$ whenever $\varepsilon_k(p) < \varepsilon < \varepsilon_{k-1}(p)$.

- Each $\varepsilon_k(p)$ is strictly increasing in p.
- Cutoff for the complete network:

$$\varepsilon_0(p) = \frac{p(2p-1)}{p + 2(1-p)^2}.$$

- Limiting cutoff for the star network:

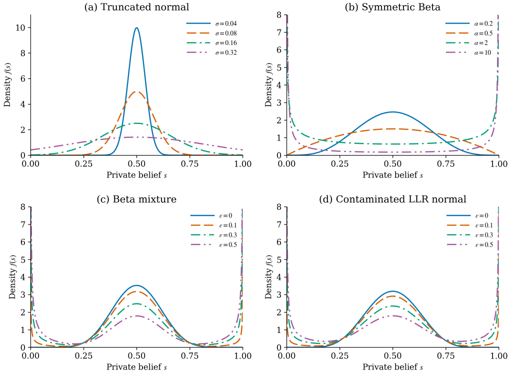
$$\varepsilon_k(p) \xrightarrow{k\to\infty} \frac{(2p-1)^2}{1+4(1-p)^2} \in (0,1).$$



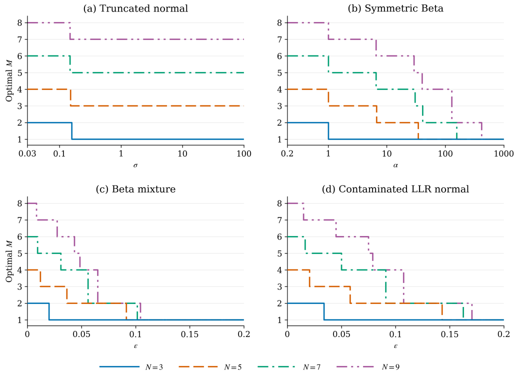
Optimal number of guinea pigs for $N = 5$

Numerical Exercises

Private-belief distributions



Optimal number of guinea pigs



Responsiveness and Overturning

- The key determinant is the **relative tail distribution** of private signals.
- **Moderately informative signals** favor preserving many guinea pigs.
  - Preserving the responsiveness of each action is then particularly valuable.
- **Frequent extreme signals** make overturning more valuable.
  - Extreme signals occur often enough to be identified through overturning histories.
- **Intermediate networks can strictly outperform both endpoints**.
  - Under smooth distributions, odd values of M can also be optimal.

Conclusion

Conclusion

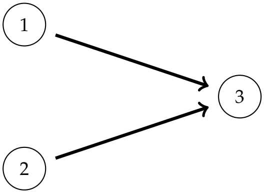
- **The efficiency of networks depends critically on the informational environment.**
  - Any network among predecessors can be uniquely optimal if N observes all predecessors.
  - The same characterization holds for positively weighted social welfare.
  - A predecessor link has an ambiguous social effect by changing the information externality.
- We identify a **trade-off** shaping the efficiency of information aggregation.
  - Sparse networks preserve **responsiveness** to private signals.
  - Dense networks identify informative signals through the **overturning effect**.
  - The **relative tail distribution** determines the balance between the two effects.

The End.

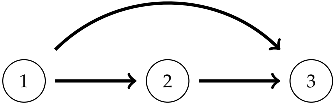
Appendix

Illustrative Examples: Three-Agent Networks

Star network



Complete network



- **Star network**: agents 1 and 2 act only on their private signals.
- **Complete network**: agent 2's action also responds to agent 1's action.
- If signals (not actions) were observable, both networks would inform agent 3 equally.

Example 1: Responsiveness Effect

Binary information structure

$$\pi(h \mid H) = \pi(l \mid L) = 9/10 \text{ and } \pi(l \mid H) = \pi(h \mid L) = 1/10.$$

- In the star network, a_1 and a_2 perfectly reveal s_1 and s_2.
- In the complete network, agent 2 may follow agent 1 and ignore s_2.
  - Hence, agent 2's distinct private signals can lead to the same action.
  - At $(s_1, s_2, s_3) = (l, h, h)$, $a_3 = 1$ in the star network but $a_3 = 0$ in the complete network.
- The star network gives agent 3 a strictly higher expected payoff.
- Intuition: Sparse networks preserve **responsiveness** of actions to private signals.

Example 2: Overturning Effect

Ordinary and conclusive positive signals

$S = \{l, h, h^*\}$, where h^* perfectly reveals H.

- In the star network, action 1 pools the signals h and h^*.
- In the complete network, agent 2 conditions on agent 1's action.
  - After $a_1 = 0$, only h^* induces $a_2 = 1 \implies (a_1, a_2) = (0, 1)$ reveals the conclusive signal.
- The complete network gives agent 3 a strictly higher expected payoff.
- Intuition: Dense networks identify strong signals from **overturning** histories.

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