Feasible Search Behavior

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Introduction

Pandora's Problem

- An agent sequentially searches for the best alternative. (e.g., shopping and job search)
- Weitzman 's (1979) search model (**Pandora's problem**) captures this scenario.
  - Multiple **asymmetric boxes** (options) with unknown **rewards**.
  - A decision maker, **Pandora**, engages in sequential search.
  - Inspecting (opening) a box is **costly**, but it **fully** reveals the reward in it.

Research Question

- In applications, agents may not fully observe the reward by inspections.
  - Rather, they would observe **partial information** on it.
  - The underlying **information structures** would affect her search behavior.

Research Question

What search behavior can be rationalized with some information structure?

- We add incomplete information to Weitzman 's (1979) model.
- We explore an interaction between **information** and **search behavior**.

RESULTS

- Search behavior = Probabilities that each box is opened.
- Theorem 1 derives **all feasible search behaviors** arises under **some** info. structures.
  - The feasible set forms a polytope.
  - A **single information structure** rationalizes any feasible search behavior.
  - This **minimizes** Pandora's welfare among all information structures.
- Theorem 2 characterizes all pairs of search and **choice behaviors**. (same implications)
- Theorem 3 considers **price competition** among sellers of options. (same feasible set)

Implications

- Information-free prediction:
  - We can derive the **prediction set** of behavior without knowing information structures.
- Information design:
  - For any objectives over search (and choice) behaviors, **optimal info. structure** is derived.
  - It minimizes Pandora's welfare **as if the designer provides no information**.
- Partial identification:
  - Econometrician wishes to identify the primitives from observed data of search behavior.
  - We can derive the **identified set** of primitives under **minimal informational assumptions**.

LITERATURE: ROBUST PREDICTIONS/INFORMATION DESIGN

- Bayes Correlated Equilibrium:
  - Pioneered works: Bergemann and Morris (2013) and Bergemann and Morris (2016).
  - Extensive form games: Makris and Renou (2023) and Doval and Ely (2020).
- Focusing on action marginal and implications to partial identification:
  - Doval *et al.* (2024) and De Oliveira and Lamba (2025).
- Ours: Same analysis but focusing on search models.
  - Search models are complicated as information depends on past actions.
  - We derive the strongest reduction principle.

Literature: Information and Search

- The role of information in **random search**:
  - Information design: Dogan and Hu (2022) and Mekonnen *et al.* (2025).
  - Competition: Board and Lu (2018), Lyu (2023), Mekonnen and Pakzad-Hurson (2025).
- The role of information in **ordered search**:
  - Competition: Au and Whitmeyer (2023) and Au and Whitmeyer (2024).
- Ours: Feasible search (and choice) behavior (with pricing) under general asymmetry.

Literature: Empirical Works

- Identification and estimation of search costs and preferences:
  - Jolivet and Turon (2019) and Moraga-González *et al.* (2023) for example.
  - Survey: Ursu *et al.* (2025).
- Recent wave is the partial identification under weak informational assumptions: e.g.,
  - Export markets: (Dickstein and Morales, 2018)
  - Entry games: (Magnolfi and Roncoroni, 2023)
  - Prescription drug choices (Dickstein *et al.*, 2024)
  - Spatial voting (Gualdani and Sinha, 2024)
  - Migration (Porcher *et al.*, 2024)
- Ours: The identified set with weak informational assumptions in search model.

Model

WEITZMAN (1979)

- Risk-neutral decision maker (**Pandora**) and finite **boxes** $i = 1, 2, \ldots, n$.
  - Each box i contains a random **reward** $x_i \in \{\underline{x}_i, \overline{x}_i\}$ with $Prob(x_i = \overline{x}_i) = \lambda_i$ and mean μ_i.
  - It costs $c_i \in (0, \lambda_i(\overline{x}_i - \underline{x}_i))$ to **open** box i and observe some $\underbrace{\textbf{signal}\ s_i}_{\text{next slide}}$ about its reward x_i.
- Pandora's strategy determines **whether or not to open which box** at each stage.
  - She can choose one of **any opened boxes** when stopping search.
- A **search behavior** $v \in \mathbb{R}^n$ describes each probability v_i that box i is opened.

INFORMATION STRUCTURE

- A **signal** s_i is a random variable whose realization depends on x_i.
  - Upon observing s_i, Pandora forms expectation $y_i = \mathbb{E}[x_i|s_i]$.
- In our model, posterior mean $y_i = \mathbb{E}[x_i|s_i]$ is the only payoff-relevant information.
  - We model an **information structure** (of box i) as a CDF $F_i \in \Delta([\underline{x}_i, \overline{x}_i])$.
  - Some signal induces F_i if and only if F_i has mean μ_i (Blackwell, 1953).
  - Call such F_i **feasible** and let $F = (F_i)_i$.

Pandora's Optimal Strategy

- Once fix CDFs $F_1, \ldots, F_n$, our model is mathematically identical to Weitzman (1979).
- A solution z_i to the following equation is called **reservation value** of box i:

$$\underbrace{c_i}_{\text{Search cost}} = \underbrace{\int_{z_i}^{\overline{x}_i} (y_i - z_i) dF_i(y_i)}_{\text{Gain from search}}$$

Pandora's Rule (Weitzman (1979))

Search boxes in descending order of reservation values

until a posterior exceeding the reservation value of any unopened box.

Feasible Search Behavior

Definition

A search behavior v is feasible if there exists a feasible CDF profile F that induces v.

- We are interested in the set of **all feasible search behaviors** and its properties.

- Formula: Suppose that $z_1 > z_2 > \cdots > z_n$ under some CDFs $F_1, \ldots, F_n$. Then,

$$F_1(z_i-) \cdot F_2(z_i-) \ldots F_{i-1}(z_i-) \leq v_i \leq F_1(z_i+) \cdot F_2(z_i+) \ldots F_{i-1}(z_i+),$$

where $F_j(z_i-)$ is the left limit of F_j and $F_j(z_i+)$ is the right limit of F_j.

ASSUMPTION

- Once can derive the range of **feasible reservation values**:

$$\underline{z}_i = \mu_i - c_i \leq z_i \leq \overline{x}_i - \frac{c_i}{\lambda_i} = \overline{z}_i.$$

Assumption

$\overline{z} \geq \underline{z}$ where $\underline{z} = \max_i \underline{z}_i$ and $\overline{z} = \min_i \overline{z}_i$.

- Under this assumption, **any search order can be induced** by some CDFs.
- This is satisfied if search costs are small or asymmetry across boxes is weak.

Results

Notations

- An order $\pi : \underbrace{\{1, \ldots, n\}}_{order} \to \underbrace{\{1, \ldots, n\}}_{box}$ is a permutation over the set of boxes.

- Define, for each i,

$$\overline{v}_{\pi,i} = \prod_{j<i} p_{\pi(j)}, \quad p_i = 1 - \frac{c_i}{\overline{x}_i - \underline{z}},$$

$$\underline{v}_{\pi,i} = \prod_{j<i} q_{\pi(j)}, \quad q_i = 1 - \frac{\underline{z}_i - \underline{x}_i}{\underline{z} - \underline{x}_i}.$$

- Intuitively, $\overline{v}_{\pi,i}$ (resp. $\underline{v}_{\pi,i}$) is an upper (resp. lower) bound of $v_{\pi(i)}$ under π.

- To simplify the notation, we write $\overline{v}_i = \overline{v}_{\pi,i}$ and $\underline{v}_i = \underline{v}_{\pi,i}$ when $\pi(i) = i$ for each i.

Main Theorem

- **Polytope** = a convex hull of finite points.

Theorem 1

1. The set of all feasible search behaviors V^* is a **polytope**.
2. v with $v_1 \geq \cdots \geq v_n$ is a **vertex** of $V \Leftrightarrow v = (\overline{v}_1, \ldots, \overline{v}_i, \underline{v}_{i+1}, \ldots, \underline{v}_n)$ for some i.
3. A **single** CDF profile $\overline{F}_1, \ldots, \overline{F}_n$ induces all feasible search behaviors, in which
   - Pandora is **indifferent** among all search orders, and
   - Pandora's expected payoff is **minimized** among all feasible CDF profiles.

Faces

- $X \subset V$ is a **face** of V if $X = \text{argmax}_{v \in V}\, r \cdot v$ for some $r \in \mathbb{R}^n \setminus \{0\}$.

Proposition 1

Take any $r \in \mathbb{R}^n \setminus \{0\}$.

Let Π_r be the set of orders π such that

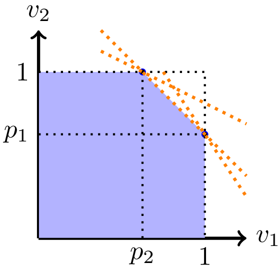
$$
\begin{aligned}
r_{\pi(1)} \cdot \frac{\overline{x}_{\pi(1)} - \underline{z}}{c_{\pi(1)}} \geq \cdots \geq\ & r_{\pi(i)} \cdot \frac{\overline{x}_{\pi(i)} - \underline{z}}{c_{\pi(i)}} \\
\geq\ & 0 \\
>\ & r_{\pi(i+1)} \cdot \frac{\underline{z} - \underline{x}_{\pi(i+1)}}{\underline{z}_{\pi(i+1)} - \underline{x}_{\pi(i+1)}} \geq \cdots \geq r_{\pi(n)} \cdot \frac{\underline{z} - \underline{x}_{\pi(n)}}{\underline{z}_{\pi(n)} - \underline{x}_{\pi(n)}}.
\end{aligned}
$$

Then, a face $X = \arg\max_{v \in V^*} r \cdot v$ is the convex hull of all points v such that,

for some $\pi \in \Pi_r$, we have $v_{\pi(j)} = \overline{v}_{\pi,j}$ for all $j \leq i$ and $v_{\pi(j)} = \underline{v}_{\pi,j}$ for all $j > i$.

Proof Sketch I: Upper Bound

- We solved information design problems with linear objectives $\sum_i r_i v_i$ for any $r_i \geq 0$.
- This provides an **upper bound** for the set of feasible search behaviors.



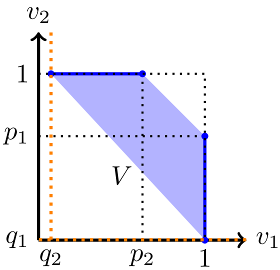
where

$$p_i = 1 - \frac{c_i}{\overline{x}_i - \underline{z}} = \max_{F_i : z_i \geq \underline{z}} F_i(\underline{z}),$$

- Intuitively, we show that making z_i closer to $\underline{z}$ increases the search length.
- Trade-off: Increasing $z_i < \underline{z}$
  - raises the probability that box i is opened. **(attraction)**
  - makes box i more attractive; Pandora is more likely to terminate the search **(persuasion)**
- Challenge: v_i is discontinuous in CDF profiles as it also depends on the search order.
- A naive approach yields an ambiguous relation between these effects.
- However, we show that, under the order $\pi \in \Pi_r$, the first effect is dominant.

Proof Sketch II: Lower Bound

- We also found a **lower bound** for each probability v_i that box i is opened.
- Then, search behaviors induced by a **pure strategy** must be in the **blue lines**.
- Therefore, feasible search behaviors must be in **their convex hull**, denoted by V.

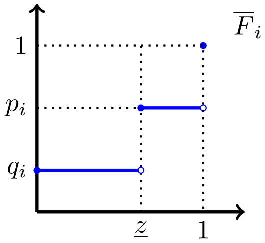


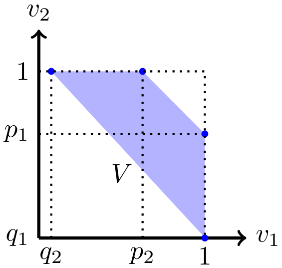
where

$$q_i = 1 - \frac{\underline{z}_i - \underline{x}_i}{\underline{z} - \underline{x}_i} = \min_{F_i : z_i \geq \underline{z}} F_i(\underline{z}-),$$

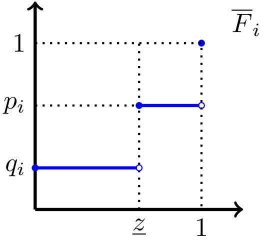
Proof Sketch III: Construction

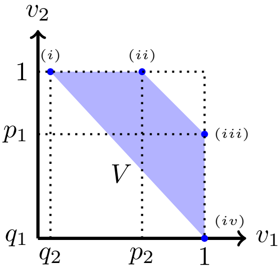
- We construct $\overline{F}$ which **implements all vertexes** of V.
- Idea: Ensures that Pandora is indifferent among as many strategies as possible.
  - Reservation value = $\underline{z}$: **All search orders are optimal**.
  - A positive (maximum) mass on $\underline{z}$: **Indifferent btw Stopping and Continuing search**.





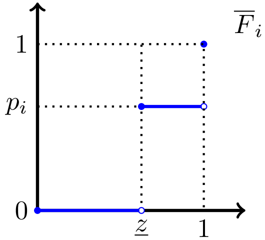
- Check that $\overline{F}$ implements all vertexes: hence all points in V.
- Under $\overline{F}_i$, there are three possible realizations: $y_i = 0$, $\underline{z}$, or 1.
  - (i) (resp. (iv)): Open box 2 (resp. 1) first and stop if y_2 (resp. y_1) is either 0 or $\underline{z}$.
  - (ii) (resp. (iii)): Open box 2 (resp. 1) first and stop if y_2 (resp. y_1) is 1.





Proof Sketch IV: Welfare

- Pandora's expected payoff is **minimized** among all information structures.
  - Consider a strategy that opens box i with $\underline{z}_i = \underline{z}$ and stops immediately.
  - Feasible under any CDFs F, **guaranteeing** payoff $\mu_i - c_i = \underline{z}_i$.
  - However, it turns out that it is a **Pandora's optimal strategy** under $\overline{F}$.



Implication I: Robust Prediction

- A metaphorical interpretation of the feasible set is about the **robust prediction**.
- In job search setting, potential workers sequentially inspect job opportunities.
- Requirement information may not perfectly reveal match quality.
- Our results: an information-free prediction for the range of possible search durations.
- This derives **unemployment periods**, which depend on the underlying information.

Implication II: Information Design

- A literal interpretation is about the **optimal information design** on search behavior.
  - **Designer** chooses F with objective function $g : \mathbb{R}^n \to \mathbb{R}$ over search behaviors.
- Amazon and Yelp design information revealed within each product page.
  - e.g., page layouts, sellers' descriptions, and customer reviews to highlight.
- Consumers learn about product quality through these designs
  - However, they cannot fully infer quality until after they purchase and consume.
- Consumers' browsing behavior affects profits through per-click advertising fees.

Proposition 2

1. $\overline{F}$ is optimal for any upper continuous objective g over search behaviors, which **minimizes** Pandora's expected payoff among all feasible CDF profile.
2. Moreover, if $g(v) = r \cdot v$ for some $r \in \mathbb{R}^n \setminus \{0\}$, then

   (i) it is optimal to let Pandora pen boxes according order in Π_r, and

   (ii) if $\overline{z} > \underline{z}$, an optimal search behavior is fully implementable.

- Our results imply that information design **adversary affects consumer welfare**.
  - An extension shows that this persist even if designer also cares eventual choice.
- Designer-optimal search order is descending in $r_i \frac{\overline{x}_i - \underline{z}}{c_i}$ if $r_i \geq 0$ for all i.

Implication III: Partial Identification

- Another one is about the **partial identification** with **weak informational assumption**.
- Suppose that analysts have access to data on search behavior v.
- They wise to estimate the primitives such as **search costs** and **preferences**.
- However, they do not know to what extent the agent learns from each inspection.
- Assumptions about information structure affects estimations and predictions.
- Our closed form characterization of the feasible set maps directly to the **identified set**.

- Let $\Gamma \subset \mathbb{R}^{4n}$ be the set of all parameters $(\overline{x}_i, \underline{x}_i, \lambda_i, c_i)$.
- **Identified set** is $I(v;\Gamma) = \{\gamma \in \Gamma \mid v \in V^*(\gamma)\}$, where $V^*(\gamma)$ is the feasible set under Γ.

Proposition 3

Take any search behavior v such that $v_1 \geq v_2 \geq \cdots \geq v_n \geq 0$.

Then, the identified set for the **symmetric parameter space** Γ^* is

$$I(v;\Gamma^*) = \left\{ \gamma \in \Gamma^* \;\middle|\; \sum_{i=1}^{n} v_i \geq 1 \text{ and } \sum_{j \leq k} v_j \leq \sum_{j \leq k} p(\gamma)^{j-1} \text{ for all } k \right\},$$

where for each $\gamma = (\overline{x}, \underline{x}, \lambda, c)$, we set $p(\gamma) = 1 - c/(\overline{x} - \underline{z})$.

- Marginal of $I(v;\Gamma^*)$ is an interval $[0, \overline{c}]$ because $p(\gamma)$ is decreasing in c.

Comparative Statics

- In any interpretations, how the feasible set varies with the primitives matters.

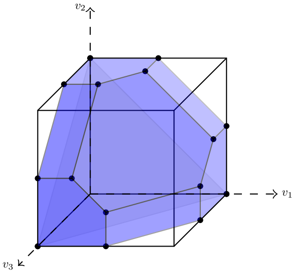
- By the closed-form expressions, we can trace how each vertex responds to changes.

Proposition 4

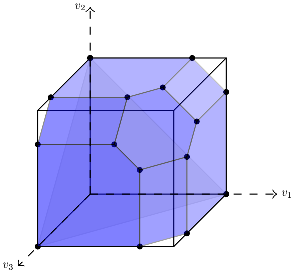
1. If $\underline{z}_i = \underline{z}$, V^* **shrinks** when λ_i increases.
2. If $\underline{z}_i < \underline{z}$, then V^* **expands** when λ_i increases.
3. In the symmetric case where $\lambda_i = \lambda$, $\overline{x}_i = \overline{x}$, $\underline{x}_i = \underline{x}$, and $c_i = c$ for all i,

   V^* **shrinks** when λ increases or c increases.

- The effect of increasing λ_i depends on the relative ex-ante attractiveness of box i.
- The effect of increasing c_i is also ambiguous.
- However, it turns out that the set is monotone in the symmetric primitives.



$\lambda = 0.8$ and $c = 0.2$.



$\lambda = 0.7$ and $c = 0.1$.

- As λ_i decreases, Pandora is more likely to sample a low value and continue searching.
- Hence, the maximum search length $r \cdot v$ weighted by $r \geq 0$ increases.
- Under the symmetric primitives, $\min r \cdot v$ does not depend on the primitives.
- Therefore, the range of $r \cdot v$ becomes larger, which suggests that V^* expands.
- If the primitives are asymmetric, $\min r \cdot v$ depend on the primitives,

  in which case V^* may neither shrink nor expand.

Extensions

Feasible Choice Behavior

- Search behavior $v \in \mathbb{R}^n$ is not a full description of Pandora's action.
- **Choice behavior** $d \in \mathbb{R}^n$: d_i is the probability that each box i is **eventually selected**.
- Question: Can we characterize the set of all **feasible pair** $(v, d) \in \mathbb{R}^n \times \mathbb{R}^n$?

Theorem 2

Suppose that $\underline{z}_i$ is homogeneous across boxes.

1. Then, the set of all feasible pairs S^* is a polytope.
2. Moreover, a single CDF profile F^* rationalizes all feasible pairs, which minimizes Pandora's expected payoff among all feasible CDF profile.

- All vertexes are derived in closed form & Implications are **identical** to Theorem 1.

Proposition 5

Suppose that $\underline{z}_i$ is homogeneous across boxes.

(v, d) with $v_1 \geq \cdots \geq v_n$ is a **vertex** of S^* if and only if, for some i and $j \leq i$, for each k,

$$v_k = \begin{cases} \overline{v}_k & \text{if} \quad k \leq i, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and } d_k = \begin{cases} \overline{v}_k - \overline{v}_{k+1} & \text{if} \quad k \leq i \text{ and } k \neq j, \\ \overline{v}_k - \overline{v}_{k+1} + p_i \cdot \overline{v}_i & \text{if} \quad k = j, \\ 0 & \text{otherwise.} \end{cases}$$

Moreover, any vertex is fully implementable.

- Each vertex consists of three elements:
  - (i) a **search order** π, (ii) a **threshold box** i, and (iii) a box j for the "**return demand**."
- F_k^* takes binary values $\{\underline{z}, h_k\}$, and Pandora uses the following strategy.
  - She opens boxes in the order π, with the highest possible probabilities to boxes up to box i.
  - Pandora stops search and selects box k whenever she observes its high realization h_k.
  - She selects box j when all realizations up to box i are low ($y_k = \underline{z}$ for all $k \leq i$).
  - Boxes after i are never opened and therefore never chosen.

Price Competition

- In consumer search, each box may be a product which is associated with its seller.
  - Consider **sellers who post prices** $t_i \geq 0$ to maximize revenue.
  - A CDF and an equilibrium pricing induce an **equilibrium search behavior**.
  - Issue: no characterization of equilibrium pricing is known.
  - However, we can still characterize equilibrium search (and choice) behaviors.

Theorem 3

Suppose that $\underline{z}_i$ is homogeneous across boxes.

1. The set of all eq. search behaviors is **same to** the set of feasible search behaviors.
2. A **single** CDF profile rationalizes all equilibrium search behaviors,

   under which the sellers obtain **zero surplus**.

General State Space

- Reward x_i may not take binary values, following general prior $F_i^0 \in \Delta([\underline{x}_i, \overline{x}_i])$.
- In this case, F_i is feasible $\Leftrightarrow$ F_i is a **mean-preserving contraction** of F_i^0, i.e.,

$$\int_{\underline{x}_i}^{x_i} F_i(y_i)dy_i \leq \int_{\underline{x}_i}^{x_i} F_i^0(y_i)dy_i \text{ for all } x_i \text{ with equality at } x_i = 1.$$

Proposition

Suppose that $\underline{z}_i$ is homogeneous across boxes.

1. The set of all search (and choice) behaviors is a **polytope**.
2. A **single** CDF profile rationalizes all search (and choice) behaviors, which **minimizes** Pandora's expected payoff among all feasible info. structures.

Conclusion

Conclusion and Remarks

- Question: Which search behavior is rationalized in search markets?
  - Add an element of **incomplete information** in Weitzman 's (1979) model.
- Results: Deriving the set of all feasible search behaviors.
  - A **welfare-minimizing** CDF profile rationalizes any point in the feasible set.
  - Generalized to the set of all feasible **pairs** of search and choice behaviors.
  - We can also extend to include **price competitions** among sellers into the mdoel.

The End.

Appendix

Feasible Set under $n = 3$

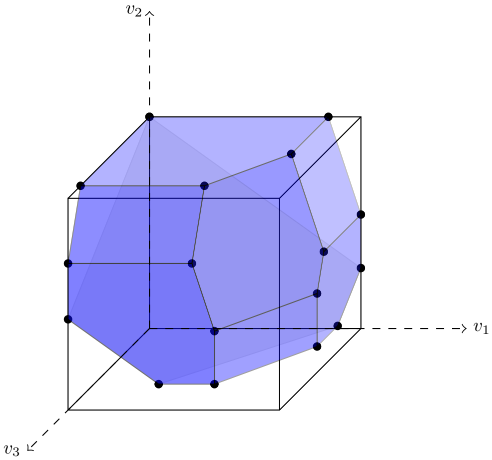


Figure: The set of feasible search behaviors V^*.

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