Value of Information in Social Learning

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Introduction

Information in Society

- An information source is **more valuable than** another if it allows better decisions.
- Blackwell (1953) evaluates information structures based on whether a **single** agent would always prefer one over another, **regardless of his/her preferences**.
- However, in the society, agents acquire information not only from their private signals **but also from the observed actions of others** (information externality).
- Comparing information only for a single decision maker is not sufficient in society.

Motivating Example: Online Shopping

- Buyers learn about product quality from **advertisements** or **product details**.
- They also have access to **the volume of past purchases** or **popularity**.
- When deciding whether to purchase a product, buyers use both information.
- Platform wishes to assess advertisements or product pages for their efficiency.
- Informational externalities created by the observability of past purchase are essential.

What We Do and Discover

Research Question

When is one information structure more socially valuable than another?

- We extend **comparison of experiments** to the **social learning model** (Banerjee, 1992; Bikhchandani, Hirshleifer and Welch, 1992; Smith and Sørensen, 2000).
- We investigate the **"socially better" information** under social learning.
- Our results characterize our binary relation and provide a simple sufficient condition.

Related Literature: Blackwell Comparison

- Pioneered works: Blackwell (1951, 1953)
  - Restricted decision problems/information structures: Lehmann (1988); Persico (2000); Athey and Levin (2018); Ben-Shahar and Sulganik (2024)
- Game-theoretic settings: Lehrer, Rosenberg and Shmaya (2010, 2013); Gossner (2000); Pęski (2008); Cherry and Smith (2012); Bergemann and Morris (2016); de Oliveira (2018)
- Repeated i.i.d. samples: (Stein (1951); Torgersen (1970); Moscarini and Smith (2002); Azrieli (2014); Mu, Pomatto, Strack and Tamuz (2021))
- Dynamic information structures in sequential decision problems: (Greenshtein (1996); de Oliveira (2018); Renou and Venel (2024); Whitmeyer and Williams (2024))

Related Literature: Social Learning

- Seminal works: Banerjee (1992), Bikhchandani *et al.* (1992), Smith and Sørensen (2000)
- The central question is whether agents can eventually learn the true state:
  - Network structure/limited observation of past actions:
    Çelen and Kariv (2004); Acemoglu, Dahleh, Lobel and Ozdaglar (2011); Lobel and Sadler (2015); Arieli and Mueller-Frank (2019); Kartik, Lee, Liu and Rappoport (2024); Xu (2023)
    (+ Banerjee and Fudenberg (2004); Gale and Kariv (2003); Callander and Hörner (2009); Smith and Sorensen (2013))
  - Costly observation of past actions: Kultti and Miettinen (2006, 2007); Song (2016)
  - Costly acquisition of private signals: Mueller-Frank and Pai (2016); Ali (2018)
- Recently, some studies examine information design in social learning.
  (Lorecchio (2022); Arieli, Gradwohl and Smorodinsky (2023); Parakhonyak and Vikander (2023); Arieli, Babichenko and Hann-Caruthers (2024))

Model

Set Up

- There is a countably infinite sequence of **agents** $i = 1, 2, \dots$ arriving sequentially.
- There are **binary state** $\Omega = \{L, H\}$ where $\mu_0 \in (0, 1)$ be the **prior** of $\omega = H$.
  - We can extend into finite state space.
- **Decision problem** $\mathcal{D}$ consists of **action set** A and **payoff function** $u : A \times \Omega \to \mathbb{R}$.
  - We assume that action set A is finite.
  - There is **no payoff externality**.
  - Moreover, we assume that all agents are **homogeneous**.

TIMING

- Period 0: Nature determines state $\omega \in \Omega$.
- Period i:
  - Agent i observes a **history** (a sequence of actions taken by agents $1, 2, \ldots, i-1$).
    - Past signal realizations are not observable.
  - Agent i receives a **private signal** $s \in S$ from **information structure** $\pi : \Omega \to \Delta(S)$ (**iid**).
    - We assume that signal space S is finite.
    - Let $\mu \in \Delta([0,1])$ be the private belief distribution induced by π.
    - $x \in \text{supp}(\mu)$ specifies the private belief about $\omega = H$ after observing private signal from π.
  - Agent i forms **posterior belief** from history and private signal and chooses action $a \in A$.
    - Strategy is a mapping $\sigma_i : A^{i-1} \times S \to \Delta(A)$, let $\boldsymbol{\sigma} = (\sigma_i)_i$ be its profile.

Analysis

Blackwell Order

- Let $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*)$ be the expected payoff of agent i under the equilibrium $\boldsymbol{\sigma}^*$ in $(\mathcal{D}, \pi)$.

Definition 0

π is **Blackwell more informative** than π' if $V_1^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) \geq V_1^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**})$ for **any** $\mathcal{D}$, any equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D}, \pi)$, and any equilibrium $\boldsymbol{\sigma}^{**}$ under $(\mathcal{D}, \pi')$.

- Blackwell order requires that agent 1 prefers π over π' for **all** decision problems.
- Denote $\pi \succsim_B \pi'$ when π is Blackwell more informative than π'.
  - $\pi \succsim_B \pi' \iff \pi'$ is a **garbling** of $\pi \iff \mu$ is a **mean-preserving spread** of μ'.

More Socially Valuable

Definition 1

π is **more socially valuable** than π' if for **all** $\mathcal{D} = (A, u)$, $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) \geq V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**})$ for **any** agent i, **any** equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D}, \pi)$, and **any** equilibrium $\boldsymbol{\sigma}^{**}$ under $(\mathcal{D}, \pi')$.

- Our binary relation requires that **all** agents prefer π over π' for **all** decision problems.
- Denote $\pi \succsim_S \pi'$ when π is more socially valuable than π'.
- By definition, $\succsim_S$ is weakly stronger than $\succsim_B$ by Pareto criterion.
- Worst equilibrium vs Best equilibrium (we discuss in the final section).

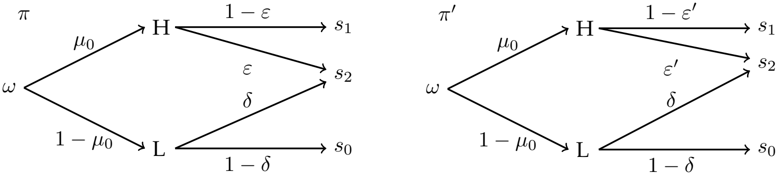
Relation with Blackwell Order

Proposition 1

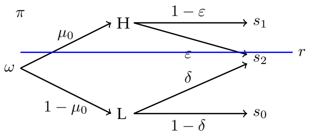
$\succsim_S$ is a **strictly stronger** binary relation than $\succsim_B$.

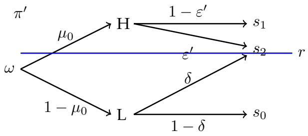
- **"Better" information (in the sense of Blackwell) may be worse for some agents**.
- This is because history provides **coarser** information than past signal realizations.
- $\succsim_S$ requires the **joint** value of history and the private signal increases.
  - If agent 1 chooses "better" action and agent 2's private signals become Blackwell more informative, agent 2's expected payoff may decrease even though they are homogeneous.
- If agents could observe **past signals instead of actions**, $\pi \succsim_B \pi'$ implies $\pi \succsim_S \pi'$.

Example I

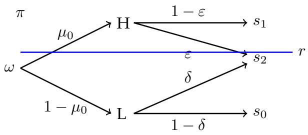


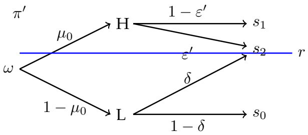
- π and π' disclose ternary signals s_0, s_1, and s_2.
- s_0 (resp. s_1) is sent only at state L (resp. H), called **conclusive signal about** state ω.
- s_2 induces intermediate private belief exceeding μ_0 by imposing $\varepsilon, \varepsilon' > \delta > 0$.
- Assume that $\varepsilon' > \varepsilon$, and thus $\pi \succsim_B \pi'$.





- $\mathcal{D}: A = \{a_0, a_1\}$, $u(a_0, L) = u(a_0, H) = 0$, $u(a_1, H) = 1 - r$, and $u(a_1, L) = -r$
- a_0 is a default action inducing constant payoff 0 for all states.
- a_1 is a risky action inducing payoff $1 - r$ at state H and $-r$ at state L.
- Optimal action is a_1 if posterior belief exceeds r and a_0 otherwise.
- Assume $\underbrace{\frac{\mu_0 \varepsilon}{\mu_0 \varepsilon + (1-\mu_0)\delta}}_{\substack{\text{Private belief} \\ \text{by } s_2 \text{ under } \pi}} < r < \min\Big\{ \underbrace{\frac{\mu_0 \varepsilon'}{\mu_0 \varepsilon' + (1-\mu_0)\delta}}_{\substack{\text{Private belief} \\ \text{by } s_2 \text{ under } \pi'}} , \underbrace{\frac{\mu_0 \varepsilon^2}{\mu_0 \varepsilon^2 + (1-\mu_0)\delta^2}}_{\substack{\text{Posterior belief} \\ \text{by } s_2 \text{ twice under } \pi}} \Big\}.$





- (Unique) Equilibrium $\boldsymbol{\sigma}^*$ under π and $\boldsymbol{\sigma}^{**}$ under π':
  - Under π, agent i chooses a_1 if and only if $s = s_1$ or at least one predecessor chooses a_1.
  - Under π', agent i chooses a_0 if and only if $s = s_0$ or at least one predecessor chooses a_0.
- $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) = \mu_0(1-\varepsilon^i)(1-r) < V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**}) = \mu_0(1-r) - (1-\mu_0)\delta^i r$ for all $i \geq 2$.
- This is because, agent 2 is optimal to choose a_1 when observing s_2 twice.
  - However, under π, a_0 is optimal when agent 1 chooses a_0 and $s = s_2$.
  - Under π', agent 2 behaves as if she can observe the past signal realization.
  - If past signals are observable, π and π' induce same payoffs for $i \geq 2$.

CHARACTERIZATION

- Let $\overline{V}_i^{\mathcal{D}}(\pi)$ be the expected payoff by observing signals from π independently i-times.

Theorem 1

$\pi \succsim_S \pi'$ if and only if $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) \geq \overline{V}_i^{\mathcal{D}}(\pi')$ for any i, $\mathcal{D}$ and equilibrium $\boldsymbol{\sigma}^*$ in $(\mathcal{D}, \pi)$.

- We construct $\bar{\mathcal{D}} = (\bar{A}, \bar{u})$ in which agents can perfectly infer past signals under π'.
  - Given $\mathcal{D} = (A, u)$, each action is **replicated** by a number of S' inducing same payoffs.
  - $\bar{A} = \{(a, k) \mid a \in A, k \in S'\}$ and $\bar{u}((a, k), \omega) = u(a, \omega)$ for all $a \in A, \omega \in \Omega$, and $k \in S'$.
  - Under π', agents chooses (a, s'_k) if $s' = s'_k$, but do not so under π (**equilibrium selection**).

Necessary Condition

- We say that π induces **unbounded beliefs** if $\mathrm{co}(\mathrm{supp}(\mu)) = [0, 1]$.

  - Smith and Sørensen (2000): in generic, asymptotic learning occurs if and only if the information structure induces unbounded beliefs.

Corollary 1

Suppose that π' is not no information.

If $\pi \succsim_S \pi'$, then π induces unbounded beliefs.

Sufficient Condition

- **Which pairs of information structures can be compared within our binary relation?**
  - Verifying sufficiency part is challenging as it depends on the underlying decision problem
- Thus, we provide a simple sufficient condition:

Theorem 2

If there exists π'' such that $\text{supp}(\mu'') = \{0, \mu_0, 1\}$ and $\pi \succsim_B \pi'' \succsim_B \pi'$, then $\pi \succsim_S \pi'$.

- If $\text{supp}(\mu'') = \{0, \mu_0, 1\}$, then π'' is some **"mixture" of full and no information**.
- With abuse, π'' is a **mixture of full and no information** if $\text{supp}(\mu'') = \{0, \mu_0, 1\}$.

Proposition 2

There exists π'' such that $\text{supp}(\mu'') = \{0, \mu_0, 1\}$ and $\pi \succsim_B \pi'' \succsim_B \pi'$ if and only if

$$1 - \sum_{s \in \text{supp}(\pi')} \min\{\pi'(s|L), \pi'(s|H)\} \leq \min\{\pi(\mu = 0|L), \pi(\mu = 1|H)\}.$$

- LHS and RHS only depend on π' and π, respectively.
  - If π is full information, RHS $= 1$.
  - If π' is no information, LHS $= 0$.
  - If π does not induce unbounded beliefs, RHS $= 0$.
- $\pi \succsim_S \pi'$ if π assigns high probability to disclose conclusive signals about both states.

Proof Sketch I

- Lemma 7: Show $\pi'' \succsim_B \pi' \Rightarrow \pi'' \succsim_S \pi'$ **if π'' is a mixture of full and no information**.
  - Lemma 6 shows that, under π'', all agents can achieve the expected payoff **as if** they were able to observe past signals **for any** decision problem and **any** equilibrium.
  - An important (equilibrium) strategy profile under π'' is given by:
    - **If agent i receives the conclusive signals, then she chooses optimal actions**.
    - **If agent i receives the uninformative signal, then she mimics agent $i-1$'s action**.
  - This is an equilibrium and yields the lower bound and upper bound of payoffs.

- (Continued.)
  - If $\pi'' \succsim_B \pi'$, then Blackwell theorem implies that $\pi''^{\otimes i} \succsim_B \pi'^{\otimes i}$.
    - Thus, we have $\overline{V}_i^{\mathcal{D}}(\pi'') \geq \overline{V}_i^{\mathcal{D}}(\pi')$ holds for all i and all $\mathcal{D}$.
  - **Past signals are Blackwell more informative than history** (Lemma 5).
    - Thus, $\overline{V}_i^{\mathcal{D}}(\pi') \geq V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**})$ for any i, any $\mathcal{D}$, and any equilibrium $\boldsymbol{\sigma}^{**}$ under π'.
  - $V_i^{\mathcal{D}}(\pi'', \boldsymbol{\sigma}^*) = \overline{V}_i^{\mathcal{D}}(\pi'') \geq \overline{V}_i^{\mathcal{D}}(\pi') \geq V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**})$ for any equilibrium $\boldsymbol{\sigma}^*$ and $\boldsymbol{\sigma}^{**}$. $\square$

Proof Sketch II

- Lemma 8: Show $\pi \succsim_B \pi'' \Rightarrow \pi \succsim_S \pi''$ **if π'' is a mixture of full and no information**.
  - We modify an equilibrium strategy profile under π as follows:
  - Any single deviation from equilibrium strategy decreases her payoff by the optimality.
  - In particular, take a strategy in which agent i behaves **as if she observes π''** than π.
  - As π'' is a mixture of full and no information, **agent i chooses optimal actions under conclusive signals and mimics agent $i-1$'s action under the uninformative signal**.

- (Continued.)
  - Given this, we further modify agent $i-1$'s strategy to the same one.
  - This decreases agent $i-1$'s payoff and **agent i's (conditional) payoff by mimicking**.
    - This is because the private belief under the uninformative signal coincides with the prior.
    - Agent i's payoff $=$ $\begin{array}{c}\text{Probability of}\\\text{conclusive signals}\\\text{for each state}\end{array} \times \begin{array}{c}\text{Payoff from}\\\text{full information}\end{array} + \begin{array}{c}\text{Probability of}\\\text{uninformative signal}\end{array} \times$ Agent $i-1$'s payoff.
  - Repeating this provides the **lower bound** of **any equilibrium payoffs**, which **coincides with** any equilibrium payoff under π''. $\square$

- This is the proof sketch of Theorem 2.
- We find and utilize the properties behind the mixtures of full and no information.
- This avoids the complicated formula of expected payoffs for all agents.
  - Lemma 7 uses an **upper bound** of payoffs under π'.
  - Lemma 8 uses a **lower bound** of payoffs under π.
  - Exact expected payoff is derived only under the mixtures of full and no information.
  - This is easy as this coincides with the payoff under observable signal setting (Lemma 6).

Discussion

Discussion I: Long-Run Comparison

- Remember that $\succsim_S$ requires **all** agents prefer π over π' for all decision problems.

Definition 2

π is **eventually more socially valuable** than π' if there is $N \in \mathbb{N}$ such that for **all** $\mathcal{D}$, $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) \geq V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**})$ for **all** $i \geq N$, **any** equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D}, \pi)$, and **any** equilibrium $\boldsymbol{\sigma}^{**}$ under $(\mathcal{D}, \pi')$.

- This requires that **all asymptotic** agents prefer π over π' for all decision problems.
- Denote $\pi \succsim_{ES} \pi'$ when π is eventually more socially valuable than π'.

Theorem 3

$\pi \succsim_{ES} \pi'$ if and only if there exists $N \in \mathbb{N}$ such that $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) \geq \overline{V}_i^{\mathcal{D}}(\pi')$ for any $\mathcal{D}$, all $i \geq N$, and any equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D},\pi)$.

Corollary 2

Suppose that π' is not no information.

If $\pi \succsim_{ES} \pi'$, then π induces unbounded beliefs.

- By Example 1, inducing unbounded beliefs is not sufficient for $\succsim_{ES}$.

- We can highlight the difference between $\succsim_{ES}$ and the eventual Blackwell order:

Definition 3: Mu, Pomatto, Strack and Tamuz (2021)

π is **eventually Blackwell more informative** than π', denoted $\pi \succsim_{EB} \pi'$,

if if there exists N such that $\pi^{\otimes i} \succsim_B \pi'^{\otimes i}$ for all $i \geq N$.

Proposition 3

$\succsim_{ES}$ is a **strictly stronger** binary relation than the eventual Blackwell order $\succsim_{EB}$.

- This mirrors the relationship between $\succsim_S$ and the Blackwell order $\succsim_B$.

- By utilizing the large sample order, we can derive a sufficient condition for $\succsim_{ES}$.

Theorem 4

If there exists π'' with $\text{supp}(\mu'') = \{0, \mu_0, 1\}$ and $\pi \succsim_{EB} \pi'' \succsim_{EB} \pi'$, then $\pi \succsim_{ES} \pi'$.

- This condition is weaker than that in Theorem 2.

- Note that $\succsim_{ES}$ is neither stronger nor weaker than $\succsim_{B}$.

- A more weaker notion only concerns **limit** welfare.

Definition 4

π is **eventually more socially valuable in the limit** than π', denoted $\pi \succsim_{LES} \pi'$, if $\lim_{i\to\infty} V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) \geq \lim_{i\to\infty} V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**})$ for **all** $\mathcal{D}$, **any** equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D}, \pi)$, and **any** equilibrium $\boldsymbol{\sigma}^{**}$ under $(\mathcal{D}, \pi')$.

Theorem 4

Suppose that π' is not no information.

$\pi \succsim_{LES} \pi'$ if and only if π induces unbounded beliefs.

$$\begin{tikzcd} \succsim_S \arrow[r] \arrow[d] & \succsim_{ES} \arrow[r] \arrow[d] & \succsim_{LES} \\ \succsim_B \arrow[r] & \succsim_{EB} & \end{tikzcd}$$

- Arrow from P to Q indicates that P is (strictly) stronger than Q.
- $\succsim_B$ (resp. $\succsim_{EB}$) is the (eventual) Blackwell order.
- $\succsim_S$ is the socially valuable order.
- $\succsim_{ES}$ (resp. $\succsim_{LES}$) is the (resp. limit) eventually socially valuable order.

Discussion II: Equilibrium Selection

- $\succsim_S$ is too strong regarding equilibrium selection rule.

Proposition 5

$\pi \succsim_S \pi$ if and only if $\mathrm{supp}(\mu) \subseteq \{0, \mu_0, 1\}$.

- Our binary relation is not a partial order by strong equilibrium selection.

- Another plausible definition:

Definition 4

π is **weakly more socially valuable** than π' if for **any** $\mathcal{D}$ and **any** equilibrium $\boldsymbol{\sigma}^{**}$ under π', there **exists** an equilibrium $\boldsymbol{\sigma}^{*}$ under π such that $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) \geq V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**})$ for **all** i.

- Denote $\pi \succsim_W \pi'$ when π is weakly more socially valuable than π'.
  - Clearly, $\succsim_W$ is a partial order.
  - By Example 1, we can show that $\succsim_W$ is **strictly stronger than** $\succsim_B$.
  - We find that $\succsim_S$ is **strictly stronger than** $\succsim_W$.
- Unfortunately, we cannot obtain a characterization and a weaker sufficient condition.

CONCLUSION

Conclusion and Remarks

- We analyze the **comparison of experiments** in the **social learning model**.
  - Investigating "better" information for **all** agents under social learning.
- We characterize our binary relation and provide a simple sufficient condition.
  - Blackwell-better information may be worse for some agents.
  - Inducing **unbounded beliefs** is necessary for **all** agents to be better off.
  - A sufficient condition is derived by utilizing **mixtures of full and no information**.
- Our results can be extended to the long-run comparison.
- Limitation: characterizing a weaker binary relation about equilibrium selection.

The End.

Appendix

Equilibrium

- Let $\alpha^{\omega}_{\leq i}(\pi, \boldsymbol{\sigma}) \in \Delta(A^i)$ be the distribution of actions taken by agents $1, 2, \ldots, i$ at ω, i.e.,

$$\alpha^{\omega}_{\leq i}(\boldsymbol{a}|\pi, \boldsymbol{\sigma}) = \sum_{(s_1,\ldots,s_i)\in S^i} \prod_{k=1}^{i} \pi(s_k|\omega)\sigma_k(a_k|a_1, \ldots, a_{k-1}, s_k).$$

- Similarly, let $\alpha^{\omega}_{i}(\pi, \boldsymbol{\sigma}) \in \Delta(A)$ be the distribution of actions taken by agent i at ω, i.e.,

$$\alpha^{\omega}_{i}(a|\pi, \boldsymbol{\sigma}) = \sum_{(a'_1,\ldots,a'_{i-1})\in A^{i-1}} \alpha^{\omega}_{\leq i}(a'_1, \ldots, a'_{i-1}, a|\pi, \boldsymbol{\sigma}).$$

- Let $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma})$ be the ex-ante expected payoff for agent i under $(\mathcal{D}, \pi)$ with $\boldsymbol{\sigma}$, i.e.,

$$V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}) = \mathbb{E}_\omega \left[\sum_{a\in A} \alpha^{\omega}_{i}(a|\pi, \boldsymbol{\sigma}) u(a, \omega) \right].$$

Equilibrium

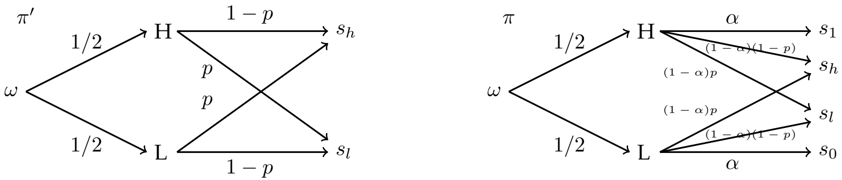
- We say that the strategy profile $\boldsymbol{\sigma}^*$ is a Bayes-Nash equilibrium under $(\mathcal{D}, \pi)$ if

$$V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) \geq V_i^{\mathcal{D}}(\pi, (\sigma_i, \sigma_{-i}^*))$$

  for all σ_i and i.

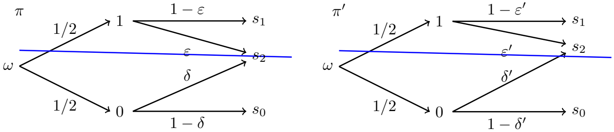
Example for Proposition 2

- Suppose that prior is uniform, that is, $\mu_0 = 1/2$.



- π' is a symmetric binary signal parameterized by $p \in [0, 1/2]$.
- π is a mixture of π^{full} and π' with probability $\alpha \in (0, 1)$.
- If $\alpha \geq 1 - 2p$, then $\pi \succsim_S \pi'$.

Example for Weak Order



- $\mathcal{D}: A = \{a_0, a_1, a_2\}, u(a_0,\omega) = u(a_2,\omega) = 0, u(a_1, H) = 1-x, u(a_1, L) = -x, x = \frac{\varepsilon}{\varepsilon+\delta}$.
- $V_2^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) = \frac{1}{2}(1-\varepsilon^2)(1-x) < V_2^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**}) = \frac{1}{2}(1-x) - \frac{1}{2}\delta'^2 x$
  - $\boldsymbol{\sigma}^*$: Under π, agent 1 chooses a_0 if $s = s_0$ or s_2 and a_1 if $s = s_1$.
  - $\boldsymbol{\sigma}^{**}$: Under π', agent 1 chooses a_0 if $s = s_0$, a_2 if $s = s_2$, and a_1 if $s = s_1$.

Proposition 4

Suppose $\pi \succsim_B \pi'$ and $\text{supp}(\mu) = \{0, x, 1\}$ and $|\mu_0 - x| \geq |\mu_0 - y|$ for all $y \in \text{supp}(\mu') \cap (0, 1)$, where $x, y \in [0, 1]$. Then, $\pi \succsim_W \pi'$

- If $x \neq \mu_0$, the equilibrium property is totally different.
- We directly construct an equilibrium under π.

Reference I

ACEMOGLU, D., DAHLEH, M. A., LOBEL, I. and OZDAGLAR, A. (2011). Bayesian learning in social networks. *The Review of Economic Studies*, **78** (4), 1201–1236.

ALI, S. N. (2018). Herding with costly information. *Journal of Economic Theory*, **175**, 713–729.

ARIELI, I., BABICHENKO, Y. and HANN-CARUTHERS, W. (2024). The positive effect of garbling on social learning.

—, GRADWOHL, R. and SMORODINSKY, R. (2023). Herd design. *American Economic Review: Insights*, **5** (4), 460–476.

— and MUELLER-FRANK, M. (2019). Multidimensional social learning. *The Review of Economic Studies*, **86** (3), 913–940.

ATHEY, S. and LEVIN, J. (2018). The value of information in monotone decision problems. *Research in Economics*, **72** (1), 101–116.

AZRIELI, Y. (2014). Comment on "the law of large demand for information". *Econometrica*, **82** (1), 415–423.

BANERJEE, A. and FUDENBERG, D. (2004). Word-of-mouth learning. *Games and Economic Behavior*, **46** (1), 1–22.

BANERJEE, A. V. (1992). A simple model of herd behavior. *The Quarterly Journal of Economics*, **107** (3), 797–817.

BEN-SHAHAR, D. and SULGANIK, E. (2024). New characterization of Blackwell's order for subsets of information structures. *Economics Letters*, **235**, 111515.

BERGEMANN, D. and MORRIS, S. (2016). Bayes correlated equilibrium and the comparison of information structures in games. *Theoretical Economics*, **11** (2), 487–522.

BIKHCHANDANI, S., HIRSHLEIFER, D. and WELCH, I. (1992). A theory of fads, fashion, custom, and cultural change as informational cascades. *Journal of Political Economy*, **100** (5), 992–1026.

BLACKWELL, D. (1951). Comparison of experiments. In *Proceedings of the second Berkeley symposium on mathematical statistics and probability*, University of California Press, vol. 2, pp. 93–103.

— (1953). Equivalent comparisons of experiments. *The Annals of Mathematical Statistics*, pp. 265–272.

CALLANDER, S. and HÖRNER, J. (2009). The wisdom of the minority. *Journal of Economic Theory*, **144** (4), 1421–1439.

Reference II

ÇELEN, B. and KARIV, S. (2004). Observational learning under imperfect information. *Games and Economic Behavior*, **47** (1), 72–86.

CHERRY, J. and SMITH, L. (2012). Strategically valuable information. *Available at SSRN 2722234*.

DE OLIVEIRA, H. (2018). Blackwell's informativeness theorem using diagrams. *Games and Economic Behavior*, **109**, 126–131.

GALE, D. and KARIV, S. (2003). Bayesian learning in social networks. *Games and Economic Behavior*, **45** (2), 329–346.

GOSSNER, O. (2000). Comparison of information structures. *Games and Economic Behavior*, **30** (1), 44–63.

GREENSHTEIN, E. (1996). Comparison of sequential experiments. *The Annals of Statistics*, **24** (1), 436–448.

KARTIK, N., LEE, S., LIU, T. and RAPPOPORT, D. (2024). Beyond unbounded beliefs: How preferences and information interplay in social learning. *Econometrica*, **92** (4), 1033–1062.

KULTTI, K. and MIETTINEN, P. (2006). Herding with costly information. *International Game Theory Review*, **8** (01), 21–31.

KULTTI, K. K. and MIETTINEN, P. A. (2007). Herding with costly observation. *The BE Journal of Theoretical Economics*, **7** (1), 0000102202193517041320.

LEHMANN, E. (1988). Comparing location experiments. *Annals of Statistics*, **16** (2), 521–533.

LEHRER, E., ROSENBERG, D. and SHMAYA, E. (2010). Signaling and mediation in games with common interests. *Games and Economic Behavior*, **68** (2), 670–682.

—, — and — (2013). Garbling of signals and outcome equivalence. *Games and Economic Behavior*, **81**, 179–191.

LOBEL, I. and SADLER, E. (2015). Information diffusion in networks through social learning. *Theoretical Economics*, **10** (3), 807–851.

LORECCHIO, C. P. L. (2022). Persuading crowds. *UB Economics–Working Papers, 2022, E22/434*.

MOSCARINI, G. and SMITH, L. (2002). The law of large demand for information. *Econometrica*, **70** (6), 2351–2366.

MU, X., POMATTO, L., STRACK, P. and TAMUZ, O. (2021). From Blackwell dominance in large samples to Rényi divergences and back again. *Econometrica*, **89** (1), 475–506.

Reference III

MUELLER-FRANK, M. and PAI, M. M. (2016). Social learning with costly search. *American Economic Journal: Microeconomics*, **8** (1), 83–109.

PARAKHONYAK, A. and VIKANDER, N. (2023). Information design through scarcity and social learning. *Journal of Economic Theory*, **207**, 105586.

PERSICO, N. (2000). Information acquisition in auctions. *Econometrica*, **68** (1), 135–148.

PĘSKI, M. (2008). Comparison of information structures in zero-sum games. *Games and Economic Behavior*, **62** (2), 732–735.

RENOU, L. and VENEL, X. (2024). Comparing experiments in discounted problems. *arXiv preprint arXiv:2405.16458*.

SMITH, L. and SØRENSEN, P. (2000). Pathological outcomes of observational learning. *Econometrica*, **68** (2), 371–398.

— and SORENSEN, P. N. (2013). Rational social learning by random sampling. *Available at SSRN 1138095*.

SONG, Y. (2016). Social learning with endogenous observation. *Journal of Economic Theory*, **166**, 324–333.

STEIN, C. (1951). Notes on the comparison of experiments. *University of Chicago*.

TORGERSEN, E. N. (1970). Comparison of experiments when the parameter space is finite. *Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gebiete*, **16** (3), 219–249.

WHITMEYER, M. and WILLIAMS, C. (2024). Dynamic signals. *arXiv preprint arXiv:2407.16648*.

XU, W. (2023). Social learning through action-signals. *Available at SSRN 4498779*.